POLYNOMIALS WITH ONE VARIABLE

Forms are:

$$P(x)=a_nx^n+a_{n+1}x^{n+1}+...+a_1x+a_0$$

If $a_n \neq 0$, then we say that the degree of polynomial P is *n*, and a_n is "*oldest*" coefficient.

Example: $P(x)=4x^3+6x^2-2x+7$

- this is a polynomial-level 3 and the oldest coefficient is 4
- interesting is that no x-member or the so-called free member we gets when we place 0 instead of x

$P(0)=4*0^3+6*0^2-2*0+7; P(0)=7$, [Or for polynomial $P(x)=a_nx^n+a_{n+1}x^{n+1}+...+a_1x+a_0$ is $P(0)=a_0$]

Addition and subtraction polynomials:

Example:

$$P(x)=3x^3-4x^2+6x-7$$
$$Q(x)=4x^3-2x^2+12x+3$$

$$P(x)+Q(x)=(3x^3-4x^2+6x-7)+(4x^3-2x^2+12x+3)$$
$$=\underline{3x^3}-\underline{\underline{4x^2}}+\underline{\underline{\underline{6x}}}-7+\underline{4x^3}-\underline{\underline{2x^2}}+\underline{\underline{\underline{12x}}}+3$$

[begin with the addition with the highest level and until you reach free "of " x:]

$$\boxed{=7x^3-6x^2+18x-4}$$

$$P(x)-Q(x)=(3x^3-4x^2+6x-7)-(4x^3-2x^2+12x+3)$$
$$=\underline{3x^3}-\underline{\underline{4x^2}}+\underline{\underline{\underline{6x}}}-7-\underline{4x^3}+\underline{\underline{2x^2}}-\underline{\underline{\underline{12x}}}-3$$
$$=\boxed{-x^3-2x^2-6x-10}$$

It is best to underline similar monomial !

Multiplication polynomials

Example 1: $P(x) = 2x - 3$

$Q(x) = x^2 + 4x - 7$

$$P(x) \cdot Q(x) = (2x-3) \cdot (x^2 + 4x - 7)$$

How to multiply?

Multiply the"each with each."

It is best to first determine the sign $(+ \cdot + = +,\ - \cdot - = +,\ + \cdot - = -,\ - \cdot + = -)$

Then multiply the numbers and the unknown.

Of course, $x \cdot x = x^2$, $x^2 \cdot x = x^3$, $x^2 \cdot x^2 = x^4 \ldots [\, x^m \cdot x^n = x^{m+n} \,]$

Let's go back to the task:

$$P(x) \cdot Q(x) = (2x-3) \cdot (x^2 + 4x - 7) =$$

$$(2x-3) \cdot (x^2 + 4x - 7) = 2x^3 + 8x^2 - 14x - 3x^2 - 12x + 21 = = 2x^3 + 5x^2 - 26x + 21$$

Example 2:

$A(x) = -x^2 + 4x - 7$

$B(x) = 2x^2 + 5x + 1$

$$A(x) \cdot B(x) = (-x^2 + 4x - 7) \cdot (2x^2 + 5x + 1)$$

$$= -2x^4 - 5x^3 - x^2 + 8x^3 + 20x^2 + 4x \ - 14x^2 - 35x - 7$$

$$= \boxed{-2x^4 + 3x^3 + 5x^2 - 31x - 7}$$

Division of polynomial

Remind, the first, sharing issues:

Example: $57146 : 23 = 2484$

$$\begin{array}{l} -46 \\ \hline 111 \\ -92 \\ \hline \ \ 194 \\ -184 \\ \hline \ \ \ 106 \\ \ \ -92 \\ \hline \ \ \ \ 14 \text{ –rest} \end{array}$$

We can write: $\frac{57146}{23} = 2848 + \frac{14}{23}$ or $\boxed{\frac{dividend}{divisor} = solution + \frac{rest}{divisor}}$

Try now polynomial: **Example 1:**

$$(2x^2 - 5x + 6) : (x - 2) = 2x - 1$$

$$\begin{array}{l} _{(-)}2x^2 -_{(+)}4x \downarrow \\ \hline \quad -x + 6 \\ \quad -_{+}x +_{-} 2 \\ \hline \qquad 4 \rightarrow \text{ rest} \end{array}$$

So:

$$\frac{2x^2 - 5x + 6}{x - 2} = 2x - 1 + \frac{4}{x - 2}$$

PROCEDURE

→ divide"the first with the first" $\frac{2x^2}{x} = 2x$
2x enroll in the solution
→ 2x multiply with dividers and put below $2x^2-5x$

→ change signs (in the brackets)
→ the first is always shorter and others gather
$-5x+4x=-x$
→ bring down next number 6
→ again share first with first $\frac{-x}{x} = -1$
→ multiply with dividers
→ change signs and gather

Example 2:

$$(x^3+2x^2-4x+5):(x+1)=x^2+x-5$$

$$_{(-)}x^3 \underset{(-)}{+} x^2$$

$$x^2-4x$$

$$_{(-)}x^2 \underset{(-)}{+} x$$

$$-5x+5$$

$$\underset{(+)}{-}5x \underset{(+)}{-} 5$$

$$10$$

So:

$$\frac{x^3+2x^2-4x+5}{x+1}=x^2+x-5+\frac{10}{x+1}$$

PROCEDURE

→ divide" the first with the first" $\frac{x^3}{x}=x^2$ and enroll in the solution
→ x^2 multiply with dividers and put below x^3+2x^2
→ change the sign x^3+2x^2
→ first be always"shorten" $2x^2-x^2=x^2$
→ ad 4x
→ again,"the first in the first" $\frac{x^2}{x}=x$
→ x multiply with divider
→ change the sign x^2+x
→ first shorten and -4x-4=-5x
→ ad +5
→ $\frac{-5x}{x}=-5$
→ -5·(x+1)=-5x-5
→ 5+5=10

Example 3:

$$(x^4-3x^3+2x^2+x-5):(x^2+2x-3)=x^2-5x+15$$

$$_{-}x^4 \underset{(-)}{+} 2x^3 \underset{(+)}{-} 3x^2$$

$$-5x^3+5x^2+x$$

$$\underset{(+)}{-}5x^3 \underset{(-)}{+} 10x^2 \underset{(-)}{+} 15x$$

$$15x^2-14x-5$$

$$_{(-)}15x^2 \underset{(-)}{+} 30x \underset{(+)}{-} 45$$

$$-44x+40 \rightarrow \text{ rest}$$

So: $$\frac{x^4-3x^3+2x^2+x-5}{x^2+2x-3}=x^2-5x+15+\frac{-44x+40}{x^2+2x-3}$$

Example 4:

$$(x^4-1):(x-1)=x^3+x^2+x+1$$

$$\begin{array}{l}
\underline{\overset{}{_{(-)}}x^4 \underset{(+)}{-} x^3} \\
\quad +x^3-1 \\
\quad \underline{\underset{(-)}{+}x^3 \underset{(+)}{-} x^2} \\
\qquad\quad x^2-1 \\
\qquad \underline{_{(-)}\, x^2 \underset{(+)}{-} x} \\
\qquad\qquad x-1 \\
\qquad\quad \underline{_{(-)}\, x \underset{(+)}{-} 1} \\
\qquad\qquad 0 \text{ - no rest}
\end{array}$$

So: $\dfrac{x^4-1}{x-1}=x^3+x^2+x+1$

Bézu theorem:

The rest of the sharing polynomial P (x) with (x-a) is equivalent to P (a), the value of polynomial P (x) in the point x = a. If the P (a) = 0, sharing is without the rest.

Example 1: Find the rest of division polynomial x^3-5x^2+6x-7 with $x-2$

x-2=0 then x =2 → a = 2

$$P(x)=x^3-5x^2+6x-7$$
$$P(2)=2^3-5\cdot 2^2+6\cdot 2-7$$
$$P(2)=8-20+12-7$$
$$P(2)=-7 \Rightarrow$$

The rest is –7

Example 2: Find the rest of the sharing polynomial $2x^3-5x+6$ with $x+1$

$$P(x)=2x^2-5x+6$$
$$P(-1)=2\cdot(-1)^2-5\cdot(-1)+6$$
$$P(-1)=2+5+6$$
$$P(-1)=13\Rightarrow$$

The rest is 13

Example 3: Factor the polynomial: $P(x)=x^3-6x^2+11x-6$

PROCEDURE

→ observe"free"member, it is that no x-by. Here is 6
→ it can be divided with : +1, -1, 2, -2, 3, -3, 6, -6
→ replace these numbers, while not get to P(a) = 0
→ we find that a = 1
→ devide with polynomial $(x-a)=(x-1)$

$$P(x)=x^3-6x^2+11x-6$$

for x=1

$$P(x)=1^3-6\cdot1^2+11\cdot1-6$$
$$P(1)=1-6+11-6$$
$$P(1)=0$$

$$(x^3-6x^2+11x-6):(x-1)=x^2-5x+6$$
$$\underset{(-)}{}x^3\ \underset{(+)}{-}x^2$$
$$-5x^2+11x$$
$$\underset{(+)}{-}5x^2\ \underset{(-)}{+}5x$$
$$6x-6$$
$$\underset{(-)}{}6x\ \underset{(+)}{-}6$$

This reduce the level of polynomial, and now x^2-5x+6

$$\begin{aligned}x^2-5x+6&=x^2-2x-3x+6\\&=x(x-2)-3(x-2)\\&=(x-2)(x-3)\end{aligned}$$

So: $x^3-6x^2+11x-66=(x-1)(x-2)(x-3)$

Example 4: Factor the polynomial: $P(x) = x^4 - 2x^3 - 2x^2 + 4x + 4$

For x=1 $\quad P(1) = 1^4 - 2\cdot 1^3 - 2\cdot 1^2 + 4\cdot 1 + 4 = 1 - 2 - 3 + 4 + 4$

$P(1) = 4 \neq 0$

For x= -1 $\quad P(-1) = (-1)^4 - 2\cdot(-1)^3 - 2\cdot(-1)^2 + 4\cdot(-1) + 4$

$P(-1) = 1 + 2 - 3 - 4 + 4 = 0$

$$(x^4 - 2x^3 - 3x^2 + 4x + 4) : (x + 1) = x^3 - 3x^2 + 4$$

$$\underset{(-)}{}x^4 \underset{(-)}{+} x^3$$

$$-3x^3 - 3x^2$$

$$\underset{(+)}{-} 3x^3 \underset{(+)}{-} 3x^2$$

$$4x + 4$$

$$\underset{(-)}{}4x \underset{(-)}{+} 4$$

Still looking: $P_1(x) = x^3 - 3x^2 + 4$

For x = -1 $\quad P_1(-1) = (-1)^3 - 3(-1)^2 + 4 = -1 - 3 + 4 = 0$

$$(x^3 - 3x^2 + 4) : (x + 1) = x^2 - 4x + 4$$

$$\underset{(-)}{}x^3 \underset{(-)}{+} x^2$$

$$-4x^2 + 4$$

$$\underset{(+)}{-} 4x \underset{(+)}{-} 4x$$

$$(x^3 - 3x^2 + 4) : (x + 1) = x^2 - 4x + 4$$

We know that : $\quad x^2 - 4x + 4 = (x - 2)^2$

So : $x^4 - 2x^3 - 3x^2 + 4x + 4 = (x + 1)(x + 1)(x^2 - 4x + 4)$

$= (x + 1)^2 (x - 2)^2$